SM3E HW11.2 Parametric Equations

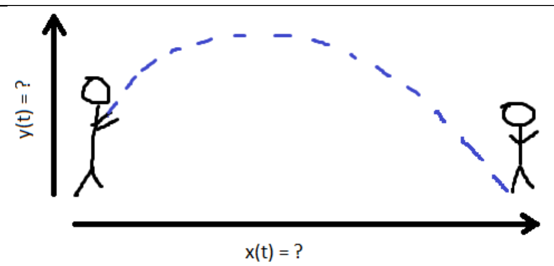
Vocab: Parametric Equation: A vector made of mathematical expressions rather than constants. Because of the freedom of the parameter, the resulting object will not always be linear…

A vector in $\mathbb{R}^n$ space is written: $v =< v_1, v_2, \ldots, v_{n-1}, v_n >$ where each v_i is a direction in an independent continuum.

A parametric equation in $\mathbb{R}^n$ space is written: $v =< v_1(t), v_2(t), \ldots, v_{n-1}(t), v_n(t) >$ where each $v_i(t)$ is an expression that gives a direction in an independent continuum and t is the domain of $v(t)$. Succinctly, parametric equations use different relations for each variable instead of allowing the variables to be directly related.

Regular Function $y = f(x)$ finds the y-values by plugging in x-values.	Parametric Function $f(x, y) =< x(t), y(t) >$ finds x-values by plugging in t-values and also finds y-values by plugging in t-values.

Example: Dave passes a basketball to Ashley. The ball's velocity toward Ashley is 3 feet per second. Dave holds the ball about 4 feet above the ground and his initial upward velocity is 12 feet per second. Unfortunately, the ball lands on Ashley's foot at ground level. Write a parametric equation that describe the motion of the ball.

Sketch the motion and determine which dimensions are required for describing the motion:	
Select expressions that fit the motion in the story.	"The ball's velocity toward Ashley is 3 feet per second." describes the $x(t)$ parameter. $x(t) = 3t$ "Dave holds the ball about 4 feet above the ground and his initial upward velocity is 12 feet per second." describes the $y(t)$ parameter. $y(t) = -16t^2 + 12t + 4$
Write the motion as a parametric equation of the form $b =< x(t), y(t) >$.	$b =< 3t, -16t^2 + 12t + 4 >$

Now that we've built a parametric equation for our story, let's answer some questions:

- When does the ball hit Ashley's foot?

When the ball hits the ground, the height (y-value) of the ball is 0. So, let's set the $y(t)$ equal to 0 and solve for t. →

$-16t^2 + 12t + 4 = 0$	Given
$4t^2 - 3t - 1 = 0$	Divide by -4
$(4t + 1)(t - 1) = 0$	Factor
$t = -\frac{1}{4}, 1$	Solve

Because the ball starts to travel at time 0, we will not include negative values of t.

$$t = 1$$

Write a sentence that answers the question: The ball hits Ashley's foot after 1 second.

- How far away from Dave is Ashley?

When the ball hits the ground, the height (y-value) of the ball is 0. This happens at $t = 1$. Let's plug $t = 1$ into $x(t)$. →

$x(t) = 3t$	Given
$x(1) = 3(1)$	Plug in 1
$x(1) = 3$	Multiply

Write a sentence that answers the question: Ashley is 3 feet away from Dave.

- Which t-values are suitable for the parametric equation?

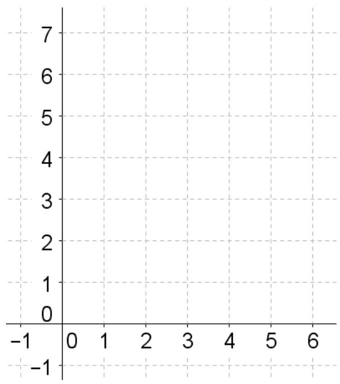
We call the moment that the ball begins to travel $t = 0$. So, any $t < 0$ won't be of use for our description of the motion of the ball. The ball hits Ashley's foot at $t = 1$. After it hits Ashley, it'd be hard to believe that the ball continued travelling into the ground, burrowing toward the center of the Earth. It'd be appropriate to end the useful t-values at $t = 1$.

Write a sentence that answers the question: Limit t such that $0 \leq t \leq 1$.

For problems 1-6, finish the parametric table and sketch the parametric curve.

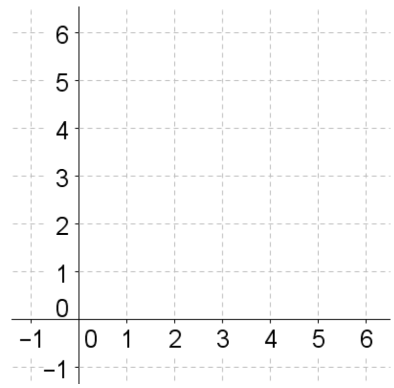
1) $a =< t+3, 4-t >$

t	$x(t)$	$y(t)$	$a = (x, y)$
-2			
-1			
0			
1			
2			



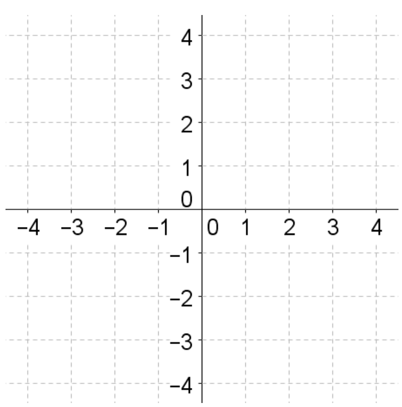
2) $b =< t^2, 3-t >$

t	$x(t)$	$y(t)$	$b = (x, y)$
-2			
-1			
0			
1			
2			



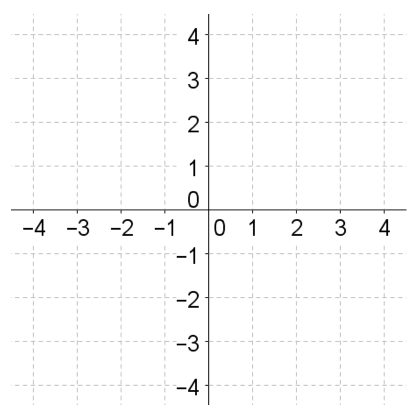
3) $c =< t-1, \frac{2}{t} >$

t	$x(t)$	$y(t)$	$c = (x, y)$
-2			
-1			
0			
1			
2			



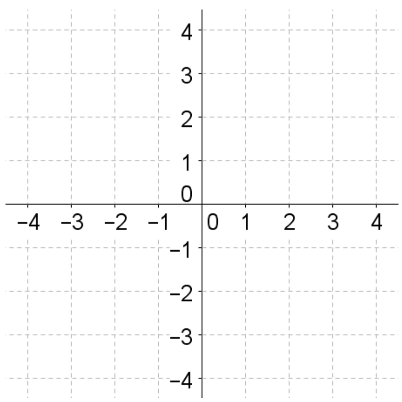
4) $d =< \sin t, \cos t >$

t	$x(t)$	$y(t)$	$d = (x, y)$
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			



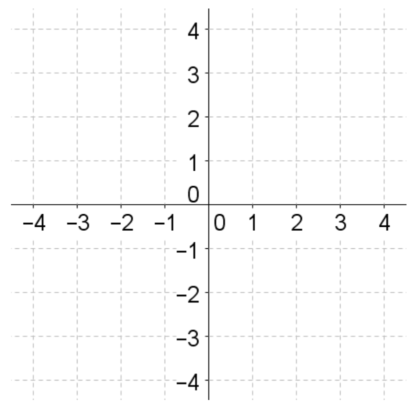
5) $f = < 1 + \sin t, 2\cos t >$

t	$x(t)$	$y(t)$	$f = (x, y)$
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			



6) $g = < \cos t, \sqrt{t} >$

t	$x(t)$	$y(t)$	$g = (x, y)$
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			

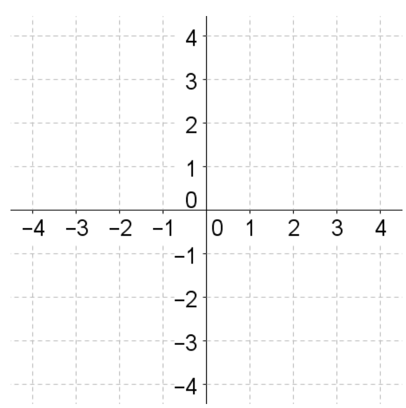


7. During a dance recital for Orem Dance Company, Tatum moves along a stage. Tatum's dance position, d, toward the right or left across the stage as a function of time is given by $x(t)$ with positive x-values indicating stage right while her positive toward or away from the audience is given by $y(t)$ with positive y-values meaning closer to the audience. t is measured in seconds after the moment she begins the dance. x, y are measured in feet.

$$d = < 2\cos(2\pi t), 3\cos(2.5\pi t) >$$

Find Tatum's position during the first two seconds of the choreography by filling out the table and sketching $d(x, y)$:

t	$x(t)$	$y(t)$	$d(x, y)$
0			
0.25			
0.5			
0.75			
1			
1.25			
1.5			
1.75			
2			



8. During a football play, $t = 0$ represents the moment that the ball is snapped, which starts the clock. Jordy is allowed to move up to 2 seconds before the ball is snapped. Jordy's motion is given by the parametric equation $j =< t^2 + 2t - 2, t - 1 >$ with $-2 \leq t \leq 2$ being the restriction on t that describes Jordy's motion.

Find Jordy's position during $-2 \leq t \leq 2$ of the play by filling out the table and sketching $j(x, y)$:

t	$x(t)$	$y(t)$	$j(x, y)$
-2			
-1			
0			
1			
2